Generation of arbitrary symmetric entangled states with conditional linear optical coupling

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An approach for generating the entangled photonic states $|\Psi_1,\Psi_2\rangle \pm |\Psi_2,\Psi_1\rangle$ from two arbitrary states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ is proposed. The protocol is implemented by the conditionally induced beam-splitter coupling which leads to the selective swapping between two photonic modes. Such coupling occurs in a quantum system prepared in the superposition of two ground states with only one of them being involved in the swapping. All the entangled states in the category, such as entangled pairs of coherent states or Fock states (N00N states), can be efficiently produced in the same way by this method.

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Bipartite symmetric entangled states refer to a generic type in the form $|\Psi_1,\Psi_2\rangle \pm |\Psi_2,\Psi_1\rangle$ up to a normalization factor. Such entangled states include the symmetric entangled coherent states (SECSs) $|\alpha,\beta\rangle \pm |\beta,\alpha\rangle$ [1] and the N00N states $|N,0\rangle \pm |0,N\rangle$ [2, 3]. Both of them have found important applications in quantum metrology; see, e.g. [4, 5]. A SECS of light fields can be transformed to a photonic Schrödinger cat state $|\gamma\rangle \pm |-\gamma\rangle$ [6] simply by a beam-splitter (BS) operation. Cat states of matter wave and even light field have been experimentally demonstrated [7–9], but a photonic one with the sufficiently large size $|\gamma|$ is still beyond the reach.

Since the seminal work of Yurke and Stoler [10], the application of Kerr nonlinearity has been suggested as the direct way to entangle light fields or construct photonic cat states [1]. Realizing strong coupling between photons via the suitable nonlinear media is, however, a rather difficult task. This barrier stimulates the parallel researches on creating the approximate states by squeezing (see, e.g. [11, 12] and the reference of [6]) and exploring the proper use of weak Kerr nonlinearity (see, e.g. [13–15]).

A less noticed problem with Kerr nonlinearity and squeezing is the availability of their single-mode versions, which are the basis for all relevant schemes thus far. A realistic photonic pulse carries multiple modes represented by the field operator $\hat{\mathcal{E}}(z) = \sum_k \hat{a}_k e^{ikz}$ (in one-dimensional space for illustration). For instance, under the action of a multi-mode self-Kerr Hamiltonian $\int dz\big(\hat{\mathcal{E}}^\dagger(z)\hat{\mathcal{E}}(z)\big)^2$ of the unit coupling constant or its equivalent form $\sum_{k1,k2,k3} \hat{a}^\dagger_{k1-k2+k3}\hat{a}_{k1}\hat{a}^\dagger_{k2}\hat{a}_{k3}$ in the wave-vector space, the output states can be significantly different from the proper ones that should have evolved under the sum of single-mode actions $\sum_k(\hat{a}^\dagger_k\hat{a}_k)^2$, even if the inputs are exactly single-mode ones. This effect of mode entanglement or mode mixing has been studied in [16, 17]. A consequence of the effect is a very limited clean cross phase (similar to that obtained from the single-mode cross-Kerr model) under highly demanding conditions [18]. On the other hand, a multi-mode squeezing action $\sum_k(\hat{a}^\dagger_k\hat{a}^\dagger_{-k} + \hat{a}_k\hat{a}_{-k})$ of one field as well deviates from its single-mode version. In contrast, the multi-mode BS Hamiltonian H_{BS} for two fields $\hat{\mathcal{E}}_1(z)$ and $\hat{\mathcal{E}}_2(z)$ takes the form $\int dz\{\hat{\mathcal{E}}^\dagger_1\hat{\mathcal{E}}_2(z) + \hat{\mathcal{E}}_1\hat{\mathcal{E}}^\dagger_2(z)\} = \sum_k(\hat{a}^\dagger_{1,k}\hat{a}_{2,k} + \hat{a}_{1,k}\hat{a}^\dagger_{2,k})$, a sum of the individual mode actions. This BS coupling enables a multi-mode photonic state to be transformed ideally like a single-mode one, because the decomposable evolution operator $\exp\{-itH_{BS}\}$ with respect to the wave-vector modes k acts independently on each mode.

In this paper we provide a method for generating arbitrary symmetric entangled states out of light fields based only on such *clean* BS coupling . Unlike the common linear optical setup, the BS coupling we need acts conditionally on the part of a superposition of quantum states at the same spatial location. Below we will show how to produce a symmetric entangled state with a conditional BS coupling and will give an example of the realization of the given type of interaction in a proper quantum system.

Protocol.— From now on, we use the term *mode* in the meaning of a single wave-vector or a frequency mode, since we will consider BS type coupling only. The two arbitrary states $|\Psi_1\rangle$, $|\Psi_2\rangle$ we will entangle are treated as the single-frequency ones.

To entangle the two states, we also need an ancilla quantum system with two stable states $|m\rangle$ and $|g\rangle$. This system can be an atom, as well as an ion, a quantum dot or a superconducting qubit. The ancilla system is initially in the state $|m\rangle$, setting the initial state for the total system as $|\Phi_0\rangle = |\Psi_1,\Psi_2\rangle\,|m\rangle$. Then we perform a σ^x rotation between the $|m\rangle$ and $|g\rangle$ and transfer the system to the superposition

$$|\Phi_1\rangle = |\Psi_1,\Psi_2\rangle\left(|m\rangle - i\,|g\rangle\right)/\sqrt{2}. \tag{1}$$

Such σ^x rotation can be realized by applying a resonant $\pi/2$ pulse to the transition $|m\rangle \to |g\rangle$.

The above superposition of $|m\rangle$ and $|g\rangle$ works as a logic control on the swapping between two input photonic modes: the swapping between the photon modes is activated in the $|g\rangle$ subspace and does not happen in the $|m\rangle$ subspace. Such conditional swapping can be realized by the BS transformation

$$\begin{pmatrix} \hat{a}_1(t) \\ \hat{a}_2(t) \end{pmatrix} = \begin{pmatrix} \cos\chi_0 t & -ie^{i\phi}\sin\chi_0 t \\ -ie^{-i\phi}\sin\chi_0 t & \cos\chi_0 t \end{pmatrix} \begin{pmatrix} \hat{a}_1(0) \\ \hat{a}_2(0) \end{pmatrix} \tag{2}$$

for the time $t_s = \pi/(2\chi_0)$, where $\hat{a}_i$ is the annihilation operator for the i-th mode and χ_0 is the effective BS coupling constant. The BS transformation can be implemented via the dispersive parametric three-wave mixing (TWM) [19, 20] or four-wave mixing (FWM) [21, 22] process. The conditional swapping results in the state

$$|\Phi_2\rangle = (|\Psi_1, \Psi_2\rangle\,|m\rangle - i\,|\Psi_2, \Psi_1\rangle\,|g\rangle)\,/\sqrt{2}. \tag{3}$$

Then, again we perform a σ^x rotation between $|m\rangle$ and $|g\rangle$ to have them transformed as $|m\,(g)\rangle \rightarrow |m\,(g)\rangle - i\,|g\,(m)\rangle$, leading to the following state

$$\begin{aligned}|\Phi_3\rangle = \frac{1}{2}[&(|\Psi_1, \Psi_2\rangle - |\Psi_2, \Psi_1\rangle)\,|m\rangle \\ &- i\,(|\Psi_1, \Psi_2\rangle + |\Psi_2, \Psi_1\rangle)\,|g\rangle\,. \end{aligned} \tag{4}$$

Finally, by measuring $|m\rangle$ and $|g\rangle$ (see the method in the following example), we make the photonic sector of the total state collapse to the target symmetric entangled states $|\Psi_1, \Psi_2\rangle \pm |\Psi_2, \Psi_1\rangle$.

A candidate for the ancilla system should satisfy two requirements. First, the quantum system should have two long-lived and well separated states between which a σ^x rotation can be performed. The second requirement is specified by the swapping stage—the system should have an appropriate energy level structure for the formation of the TWM or FWM interaction loop where two of the transitions have to be strongly coupled to the input fields. These conditions can be satisfied by certain trapped natural atoms or ions, single color centers, quantum dots or superconducting qubits based on the Josephson junctions, which have multi-level structures and can also be strongly coupled to the suitable field modes.

The idea to induce the conditional interaction of matter wave (ion or atom) state superposition with one optical mode was proposed in [23] for creating the cat states [7–9, 24]. The difference of our setup is that it allows to realize a conditional coupling directly between two photonic modes for their swapping. This is necessary for constructing a SECS $|\alpha, \beta\rangle \pm |\beta, \alpha\rangle$ with $\alpha \gg \beta$ (for making a cat state of large size) or a N00N state. Our method aims to generate all such states in a unified way.

Example.— Here the single ion of calcium $^{43}Ca^+$ trapped in ion trap and embedded in optical resonator is considered as the ancilla system. The energy level structure of the calcium ion $^{43}Ca^+$ is illustrated in Fig. 1. The research on trapped ions is reviewed in [25], and a recent experiment with the setup similar to our proposal is reported in [26].

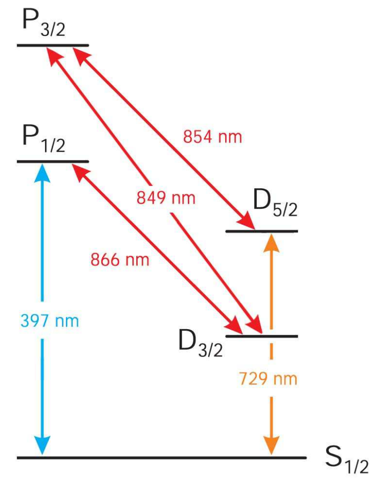

FIG. 1: (color online). $^{43}Ca^+$ energy level scheme.

The two ground states $|m\rangle$ and $|g\rangle$ we use are $4S_{1/2}\,(F=4,\ m_J=0)$ and $3D_{5/2}\,(F=6,\ m_J=0)$, respectively. These particular levels are chosen as the ground states for two reasons. First, both of the states are long-lived (up to the order of 1 s). Second, due to the selection rule and large energy difference between them, one of the ground states is excluded from a parametric FWM loop so that the conditional BS coupling can be realized. The ion transition $3D_{5/2}\,(F=6) \leftrightarrow 4P_{3/2}\,(F=5)$ at 854 nm is coupled to the σ^+ and σ^- circularly polarized modes prepared in states $|\Psi_1\rangle_{\sigma^+}$ and $|\Psi_2\rangle_{\sigma^-}$. In case when the input modes are in the coherent states the SECSs will be generated and for the Fock and vacuum inputs the NOON states will be obtained. As follows from the interaction configuration presented in Fig.2 only the ground state $|g\rangle$ $(3D_{5/2}\,(F=6,\ m_J=0))$ becomes coupled to the optical modes but there is no coupling to the optical modes for the state $|m\rangle$ $(4S_{1/2}\,(F=4,\ m_J=0))$. In order to perform a σ^x rotation between $|m\rangle$ and $|g\rangle$ and bring the ion into the superposition state in Eq. (1), a resonant $\pi/2$ laser pulse at 729 nm with π - polarization is applied to the quadrupole transition $4S_{1/2} \leftrightarrow 3D_{5/2}$. During the swapping stage there are also two classical pumping pulses with the orthogonal circular polarizations applied to the transitions $3D_{3/2}\,(F=5, m_J=0) \leftrightarrow 4P_{3/2}\,(F=5, m_J=\pm 1)$ at 849 nm, while the cavity modes σ^+ and σ^- are coupled to the transitions $3D_{5/2}\,(F=6, m_J=0) \leftrightarrow 4P_{3/2}\,(F=5, m_J=\pm 1)$ in the parametric FWM loop.

To realize the parametric BS coupling, all real transitions in the loop should be suppressed and ideally the ion should stay in its ground state during the swapping process. Therefore all fields should be highly detuned from the resonance and satisfy certain conditions (see the discussion below). By controlling the duration of the classical pulses we can control the precise parametric interaction time for obtaining the state in (3). After another $\pi/2$ laser pulse at 729 nm with π - polarization there will

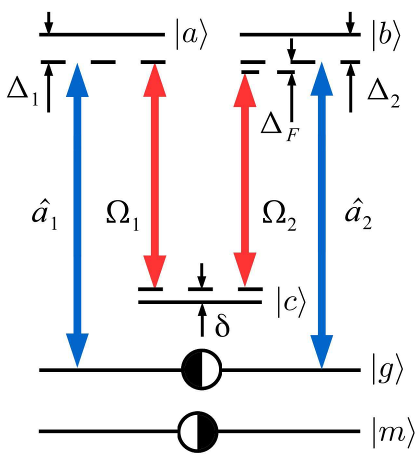

FIG. 2: (color online). Level scheme for realizing the conditional BS coupling between two photonic modes $\widehat{a}_1$ and $\widehat{a}_2$ (blue arrows). The red arrows represent the classical coupling fields. The white and black circles on levels $|g\rangle$ and $|m\rangle$ indicate that they are in a superposition and only one of them is involved into the parametric loop. The energy levels of this general scheme correspond to those of $^{43}Ca^{+}$ for our example as follows: $|m\rangle \to 4S_{1/2}\,(F=4,\ m_J=0)$; $|g\rangle \to 3D_{5/2}\,(F=6,\ m_J=0)$; $|a\rangle \to 4P_{3/2}\,(F=5,\ m_J=1)$; $|b\rangle \to 4P_{3/2}\,(F=5,\ m_J=-1)$ and $|c\rangle \to 3D_{3/2}\,(F=6,\ m_J=0)$.

be the state in (4). The detection of the ground states for the final projection onto the target states is implemented by exciting the transition $4S_{1/2} \to 4P_{1/2}$ at 397 nm; see the similar technique in [27, 28]. The presence of the fluorescence collapses the ion wave function onto $|m\rangle$ and the absence of the fluorescence indicates the state $|g\rangle$.

BS mechanism.— The dispersive FWM process for realizing the conditional swapping in our protocol can be implemented in any system with the level scheme in Fig. 2. The Hamiltonian for the process shown in Fig. 2 takes the form ($\hbar \equiv 1$)

$$\begin{aligned} H = & -\Delta_1\sigma_{aa} - \Delta_2\sigma_{bb} - \delta\sigma_{cc} + g_1\widehat{a}_1^{\dagger}\sigma_{ga} \\ & + g_2\widehat{a}_2^{\dagger}e^{-i\Delta_F t}\sigma_{gb} + \Omega_1\sigma_{ca} + \Omega_2\sigma_{cb} + h.c. \end{aligned} \tag{5}$$

in a rotation frame. Here $\sigma_{ij} = |i\rangle\langle j|$ is the atomic spin flip operator; $g_{1(2)}$ is the coupling constant; $\Delta_{1(2)} = \omega_{a1(2)} - \omega_{ga(b)}$ is the one-photon detuning, $\delta = \omega_1 + \omega_{a1} - \omega_{cg}$ is the two-photon detuning, and $\Delta_F = \omega_1 + \omega_{a1} - \omega_2 - \omega_{a2}$ is the four-photon detuning, with $\omega_{1(2)}$ being the frequency of the classical pumping pulse with the Rabi frequency $\Omega_{1(2)}$, $\omega_{a1(a2)}$ being the frequency of the input pulse. The Schrödinger equation for each energy level component of the state $|\Psi(t)\rangle$ reads:

$$i\frac{d}{dt}\langle g|\Psi(t)\rangle = g_1\widehat{a}_1^{\dagger}\langle a|\Psi(t)\rangle + g_2\widehat{a}_2^{\dagger}e^{-i\Delta_F t}\langle b|\Psi(t)\rangle \tag{6a}$$

$$\begin{aligned} i\frac{d}{dt}\langle b|\Psi(t)\rangle = & -\Delta_2\langle b|\Psi(t)\rangle \\ & + \Omega_2\langle c|\Psi(t)\rangle + g_2\widehat{a}_2 e^{i\Delta_F t}\langle g|\Psi(t)\rangle \end{aligned} \tag{6b}$$

$$\begin{aligned} i\frac{d}{dt}\langle a|\Psi(t)\rangle = & -\Delta_1\langle a|\Psi(t)\rangle \\ & + \Omega_1\langle c|\Psi(t)\rangle + g_1\widehat{a}_1\langle g|\Psi(t)\rangle \end{aligned} \tag{6c}$$

$$\begin{aligned} i\frac{d}{dt}\langle c|\Psi(t)\rangle = & -\delta\langle c|\Psi(t)\rangle + \Omega_1\langle a|\Psi(t)\rangle \\ & + \Omega_2\langle b|\Psi(t)\rangle. \end{aligned} \tag{6d}$$

The effective BS Hamiltonian for the similar dispersive FWM schemes can be derived by the time-independent perturbation method [22]. Here we apply the more general method of the adiabatic elimination [29] to show the realization of the effective BS coupling. It is important to mention that the one- and two-photon detunings should be high enough to prevent any real transition from its ground state. It is therefore possible to see the effective dynamics of the photonic modes while the system is staying in the ground state $|g\rangle$.

First, assuming that initially the system is prepared in its ground state $|g\rangle$, i.e. $\langle b|\Psi(t_0)\rangle = \langle a|\Psi(t_0)\rangle = \langle c|\Psi(t_0)\rangle = 0$, we eliminate the transitions from the state $|g\rangle$ to $|a\rangle$ and $|b\rangle$. Under this assumption we integrate Eqs. (6a) - (6c)

$$\begin{aligned} \langle g|\Psi(t)\rangle = & \langle g|\Psi(t_0)\rangle - i\int_{t_0}^{t} d\tau[g_1\widehat{a}_1^{\dagger}\langle a|\Psi(\tau)\rangle \\ & + g_2\widehat{a}_2^{\dagger}e^{-i\Delta_F\tau}\langle b|\Psi(\tau)\rangle] \end{aligned} \tag{7a}$$

$$\begin{aligned} \langle b|\Psi(t)\rangle = & -i\int_{t_0}^{t} d\tau e^{i\Delta_2(t-\tau)} \\ & \times\left[\Omega_2\langle c|\Psi(\tau)\rangle + g_2\widehat{a}_2 e^{i\Delta_F\tau}\langle g|\Psi(\tau)\rangle\right] \end{aligned} \tag{7b}$$

$$\begin{aligned} \langle a|\Psi(t)\rangle = & -i\int_{t_0}^{t} d\tau e^{i\Delta_1(t-\tau)} \\ & \times\left[\Omega_1\langle c|\Psi(\tau)\rangle + g_1\widehat{a}_1\langle g|\Psi(\tau)\rangle\right]. \end{aligned} \tag{7c}$$

Then we substitute Eq. (7a) into Eqs. (7b) and (7c) to obtain the relation

$$\langle b|\Psi(t)\rangle = \frac{\Omega_2}{\Delta_2}\langle c|\Psi(t)\rangle + \frac{g_2\widehat{a}_2 e^{i\Delta_F t}}{\Delta_2}\langle g|\Psi(t)\rangle \tag{8a}$$

$$\langle a|\Psi(t)\rangle = \frac{\Omega_1}{\Delta_1}\langle c|\Psi(t)\rangle + \frac{g_1\widehat{a}_1}{\Delta_1}\langle g|\Psi(t)\rangle \tag{8b}$$

where we use the assumption $g_i\sqrt{n_i}/\Delta_i \ll 1$ and keep only the first order of the small term, and n_i is the average photon number of the i-th input mode.

Next, in order to obtain the decoupled dynamics of the effective two-level system of $|g\rangle$ and $|c\rangle$, we substitute Eqs.(8a) and (8b) into Eqs. (6a) and (6d) and obtain

$$\frac{d}{dt}e^{i\alpha_g t}\langle g|\Psi(t)\rangle = -ie^{i\alpha_g t}\beta^{\dagger}(t)\langle c|\Psi(t)\rangle \tag{9a}$$

$$\frac{d}{dt}e^{i\alpha_c t}\left\langle c|\Psi\left(t\right)\right\rangle = -ie^{i\alpha_c t}\beta\left(t\right)\left\langle g|\Psi\left(t\right)\right\rangle, \tag{9b}$$

where we have introduced the following functions $\alpha_c = -\delta+\Omega_1^2/\Delta_1+\Omega_2^2/\Delta_2$, $\alpha_g = g_1^2\widehat{a}_1^\dagger\widehat{a}_1/\Delta_1+g_2^2\widehat{a}_2^\dagger\widehat{a}_2/\Delta_2$, and $\beta = \Omega_1 g_1\widehat{a}_1/\Delta_1+\Omega_2 g_2\widehat{a}_2 e^{i\Delta_F t}/\Delta_2$.

The dynamics of the states $|g\rangle$ and $|c\rangle$ will be decoupled further. Integrating Eq.(9b) we get the relation

$$\left\langle c|\Psi\left(t\right)\right\rangle = \frac{1}{\delta}\left[\frac{\Omega_1 g_1\widehat{a}_1}{\Delta_1}+\frac{\Omega_2 g_2\widehat{a}_2}{\Delta_2}e^{i\Delta_F t}\right]\left\langle g|\Psi\left(t\right)\right\rangle, \tag{10}$$

where we keep only the first order of the parameter $g_i\sqrt{n_i}\Omega_i/\left(\delta\Delta_i\right) \ll 1$.

Finally, substituting Eq. (10) into (9a), we obtain the decoupled evolution of the state $|g\rangle$

$$i\frac{d}{dt}\left\langle g|\Psi\left(t\right)\right\rangle = H_{eff}\left\langle g|\Psi\left(t\right)\right\rangle \tag{11}$$

where

$$H_{eff}(t) = \frac{g_1^2\widehat{a}_1^\dagger\widehat{a}_1}{\Delta_1}+\frac{1}{\delta}\left(\frac{\Omega_1 g_1\widehat{a}_1^\dagger}{\Delta_1}+\frac{\Omega_2 g_2\widehat{a}_2^\dagger}{\Delta_2}e^{-i\Delta_F t}\right) \times\left(\frac{\Omega_1 g_1\widehat{a}_1}{\Delta_1}+\frac{\Omega_2 g_2\widehat{a}_2}{\Delta_2}e^{i\Delta_F t}\right)+\frac{g_2^2\widehat{a}_2^\dagger\widehat{a}_2}{\Delta_2}. \tag{12}$$

The conditions $g_i\sqrt{n_i}/\Delta_i \ll 1$, $g_i\sqrt{n_i}\Omega_i/\left(\delta\Delta_i\right) \ll 1$ leading to the above effective dynamics prevent the one- and two-photon transitions out of a ground level and can be realized by adjusting the system parameters. For our example using ${}^{43}Ca^{+}$ with $g_{1(2)} \sim 10$ MHz, it is possible to set the Rabi frequencies $\Omega_{1(2)} \sim 1$ GHz, the one-photon detunings $\Delta_{1(2)} \sim 1$ GHz, and the two-photon detuning $\delta \sim 1$ GHz, given the average photon numbers up to $n_i = 100$. The symbol "$\sim$" means the order of the values here. The sizes $\sqrt{n_i}$ of the states to be entangled can be made larger simply by increasing the detunings.

Swapping operation.— The unitary evolution operator of the time-dependent effective Hamiltonian H_{eff} in (12) can be decomposed as

$$\mathrm{T}\exp\left\{-i\int_0^t d\tau H_{eff}(\tau)\right\} = \exp\left\{-i\eta_1\widehat{a}_1^\dagger\widehat{a}_1 t - i\eta_2\widehat{a}_2^\dagger\widehat{a}_2 t\right\} \times \mathrm{T}\exp\left\{-i\chi_0\int_0^t e^{-i\delta_F\tau}d\tau\ \widehat{a}_1\widehat{a}_2^\dagger + h.c.\right\}, \tag{13}$$

where T stands for a time-ordered operation, $\eta_i = \frac{g_i^2}{\Delta_i}+\frac{g_i^2\Omega_i^2}{\delta\Delta_i^2}$, $\chi_0 = \frac{\Omega_1\Omega_2 g_1 g_2}{\delta\Delta_1\Delta_2}$, and $\delta_F = \Delta_F+\eta_1-\eta_2$. The general form of such decomposition is given in [30]. The first of the decomposed operators in Eq.(13) is a phase shift operation and the second is a BS operation. For example, by tuning the system parameters so that the conditions $\delta_F = 0$ and $\delta = -2\Delta_i$ (assuming $g_1 = g_2$ and $\frac{\Omega_1}{\Delta_1} = \frac{\Omega_2}{\Delta_2} = 1$) are satisfied, their combined action implements an ideal swapping $\widehat{a}_1 \leftrightarrow \widehat{a}_2$ after the time t_s accumulating $|\chi_0 t_s| = 0.5\pi$. Given the data following Eq.(12) a pair of input states could be entangled within a few microseconds.

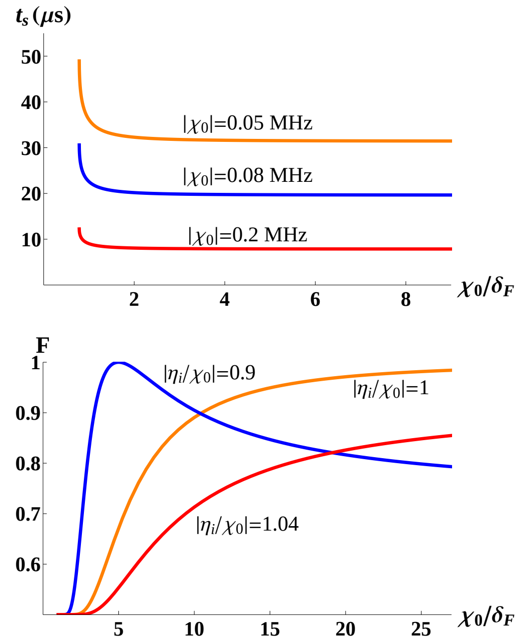

FIG. 3: (color online). Upper: (a) Relation between the swapping time and the ratio χ_0/δ_F. Lower: (b) Fidelity of the generated states with the target output $|\alpha\rangle|\beta\rangle \pm |\beta\rangle|\alpha\rangle$ with $\alpha = 6\sqrt{2}$ and $\beta = \sqrt{2}$, which can be converted to the cat states $|\gamma\rangle \pm |-\gamma\rangle$ of $\gamma = 5$. A unit fidelity can be reached in the regime $|\eta_i/\chi_0| < 1$ at a not so large ratio χ_0/δ_F.

(color online) Upper: (a) Relation between the swapping time and the ratio χ_0/δ_F. Lower: (b) Fidelity of the generated states with the target output $|\alpha\rangle|\beta\rangle \pm |\beta\rangle|\alpha\rangle$ with $\alpha = 6\sqrt{2}$ and $\beta = \sqrt{2}$, which can be converted to the cat states $|\gamma\rangle \pm |-\gamma\rangle$ of $\gamma = 5$. A unit fidelity can be reached in the regime $|\eta_i/\chi_0| < 1$ at a not so large ratio χ_0/δ_F.

In a general situation the swapping time t_s is determined by the relation $2\chi_0\sin(0.5\delta_F t_s)/\delta_F = 0.5\pi$, implying a quickly stabilized swapping time with increasing ratio χ_0/δ_F; see Fig. 3(a). Meanwhile the output state will be $|\alpha,\beta\rangle \pm |\beta e^{i\varphi_1},\alpha e^{i\varphi_2}\rangle$, where $\varphi_i = \eta_i t_s - 0.5\pi + 0.5\delta_F t_s$, if the inputs are two coherent states $|\alpha\rangle$ and $|\beta\rangle$. The fidelity of the output state is determined by the two ratios $|\chi_0/\delta_F|$ and $|\eta_i/\chi_0|$ (given $\eta_1 = \eta_2$). As it is shown in Fig. 3(b), a high-quality output state will be realized with the proper ratios that can be achieved by adjusting the system parameters.

Decoherence effect.— One limitation resulting from the decoherence in the parametric loop is that the transitions have some broadening and the fields detuning Δ_i, δ should be much larger than the bandwidth of the corresponding transitions Γ_{gi} ($i = a, b$ and c), i.e. $\Delta_i \gg \Gamma_{ga,gb}$ and $\delta \gg \Gamma_{gc}$. The other limitation comes from the possible population of the other energy levels from the ground state $|g\rangle$ during the swapping process. The radiative decay in the system occurs only after the excitation of the

system from the ground state. As we apply a dispersive interaction, the probability for the one-photon excitation scales as $P_{a,b} = g_{1,2}^2 n_{1,2}\Delta_{1,2}^{-2}$ (see Eqs. (8a) and (8b)), and the probability of the two-photon excitation scales as $P_c = |\Omega_1 g_1\sqrt{n_1}/(\Delta_1\delta)+\Omega_2 g_2\sqrt{n_2}e^{i\Delta_F t}/(\Delta_2\delta)|^2$ (see Eq. (10)). Therefore the effective decays in the system should be described by the product of the decay rate of the particular state and the probability of the electron excitation at this level. In order to have the negligible contribution from the decay processes, the swapping time should be much shorter than the effective radiative lifetime $t_s \ll 1/(\gamma_i P_i)$. For example, if one takes the swapping time for the matched four-photon detuning giving $\delta_F = 0$, there should be the condition $\gamma_{a,b} \ll \frac{2}{\pi}\frac{\Omega_1\Omega_2}{\delta}\frac{\Delta_{1,2}}{\Delta_{2,1}}\frac{g_{2,1}}{g_{1,2}}$ for neglecting the decay of the intermediate states $|a\rangle$ ($|b\rangle$) and the condition $\gamma_c \ll \frac{2}{\pi}|\sqrt{n_1 m}+\sqrt{n_2 e^{i\Delta_F t}m^{-1}}|^{-2}\delta$, where $m = \frac{\Omega_1 g_1\Delta_2}{\Omega_2 g_2\Delta_1}$, for neglecting the decay of the state $|c\rangle$. For the example using $^{43}Ca^+$ with $\gamma_{a,b} \sim 10$ MHz and $\gamma_c \sim 10$ Hz the above conditions can be easily established. These conditions guarantee the outputs of the swapping process to be pure states.

Summary.— With an example, we have illustrated how to entangle two arbitrary states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ to the symmetric form $|\Psi_1,\Psi_2\rangle \pm |\Psi_2,\Psi_1\rangle$ with an induced conditional BS type coupling that avoids the physical limitation on nonlinear couplings. In contrast to all previous schemes, the entanglement strategy is independent of the specific input states, e.g. SECSs and N00N states can be generated in the same way. The FWM process for realizing the effective BS coupling is within the current experimental technology. This approach can help to achieve the goals of entangling light fields with flexibility and creating cat states of large size.

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[1] B. C. Sanders, J. Phys. A: Math. Theor. 45, 244002 (2012).
[2] B. C. Sanders, Phys. Rev. A 40, 2417 (1989).
[3] A. N. Boto, *et al.*, Phys. Rev. Lett. 85, 2733 (2000).
[4] H. Lee, P. Kok, and J. P. Dowling, J. Mod. Opt. 49, 2325 (2002).
[5] T. C. Ralph, Phys. Rev. A 65, 042313 (2002).
[6] S. Glancy and H. M. Vasconcelos, J. Opt. B: Quantum Semiclassical Opt. 25, 712 (2008).
[7] C. Monroe, D. M. Meekhof, B. E. King, and D. J. Wineland, Science 272, 1131 (1996).
[8] M. Brune, *et al.*, Phys. Rev. Lett. 77, 4887 (1996).
[9] S. Deleglise, *et al.*, Nature (London) 455, 510 (2008).
[10] B. Yurke and D. Stoler, Phys. Rev. Lett. 57, 13 (1986).
[11] S. Song, C. M. Caves, and B. Yurke, Phys. Rev. A 41, 5261 (1990).
[12] M. Dakna, T. Anhut, T. Opatrny, L. Knoll, and D.-G. Welsch, Phys. Rev. A 55, 3184 (1997).
[13] H. Jeong, Phys. Rev. A 72, 034305 (2005).
[14] M. S. Kim and M. Paternostro, J. Mod. Opt. 54, 155 (2007).
[15] B. He, M. Nadeem, and J. A. Bergou, Phys. Rev. A 79, 035802 (2009).
[16] J. Gea-Banacloche, Phys. Rev. A 81, 043823 (2010).
[17] B. He and A. Scherer, Phys. Rev. A 85, 033814 (2012).
[18] B. He, Q. Lin, and C. Simon, Phys. Rev. A 83, 053826 (2011).
[19] R. M. Serra, C. J. Villas-Boas, N. G. de Almeida, and M. H. Y. Moussa, Phys. Rev. A 71, 045802 (2005).
[20] G.-W. Lin, X.-B. Zou, M.-Y. Ye, X.-M. Lin, and G.-C. Guo, Phys. Rev. A 77, 064301 (2008).
[21] D. D. Yavuz, Phys. Rev. A 71, 053816 (2005).
[22] A. V. Sharypov, X. Deng, and L. Tian, Phys. Rev. B 86, 014516 (2012).
[23] C. M. Savage, S. L. Braunstein, and D. F. Walls, Opt. Lett. 15, 628 (1990).
[24] C. C. Gerry and P. L. Knight, Chap. 10, Introductory Quantum Optics, Cambridge University Press (2005).
[25] H. Haeffner, C. F. Roos, and R. Blatt, Phys. Rep. 469, 155 (2008).
[26] A. Stute, *et al.*, Appl. Phys. B 107, 1147 (2012).
[27] F. Schmidt-Kaler, *et al.*, Nature (London) 422, 408 (2003).
[28] A. H. Myerson, *et al.*, Phys. Rev. Lett. 100, 200502 (2008).
[29] S. A. Gardiner, Quantum Measurements, Quantum Chaos, and Bose-Einstein Condensates, PhD Thesis (2000).
[30] B. He, Phys. Rev. A 85, 063820 (2012).